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Modular Adaptive Control of a Nonlinear Aeroelastic System

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The paper treats the question of adaptive control of prototypical aeroelastic wing sections with structural nonlinearity. The chosen dynamic model describes the nonlinear plunge and pitch motion of a wing. The model has both the plunge and pitch structural nonlinearities, and a single control surface is used for the purpose of control. It is assumed that only the sign of the coefficient of the control input and the lower bound on its absolute value are known, and the remaining parameters of the model are completely unknown. Modular adaptive control systems based on estimation-based design are derived for the control of the pitch angle and the plunge displacement. Unlike the direct adaptive controllers available in the literature, an input-to-state stabilizing control law is used herein. This control law accomplishes input-to-state stability with respect to parameter estimation error treated as a disturbance input. The control system includes a passive identifier (an observer and an adaptation law) for the parameter estimation. In the closed-loop system asymptotic stabilization of the plunge and pitch motion is accomplished. Simulation results are presented to show that the modular adaptive control systems accomplish flutter suppression in spite of the large uncertainties in the system. Simulation results also show that the closed-loop system without parameter adaptation fails to suppress flutter, but the input-to-state-stabilizing controller is effective in keeping the pitch and plunge responses bounded.

Nomenclature

a	=	nondimensionalized distance from the midchord to the elastic axis
b	=	semichord of the wing
c_h	=	structural damping coefficient in plunge caused by viscous damping
c_{ii}, κ, Γ	=	control gains and weighting matrix
c_α	=	structural damping coefficient in pitch caused by viscous damping
G_0, F, D, b_0	=	system matrices
h	=	plunge displacement
I_α	=	mass moment of inertia about the elastic axis
k_h	=	structural spring constant in plunge
k_α	=	structural spring constant in pitch
m_t	=	mass of the plunge–pitch system
m_w	=	mass of the wing
s_p	=	span
U	=	freestream velocity
V_i, V_h, V_o, V_α	=	Lyapunov functions
x, $\tilde{x}$	=	states and observation error
x_α	=	nondimensionalized distance measured from the elastic axis to the center of mass
α	=	pitch angle
β	=	flap deflection
ϵ	=	parameter used in update law
Θ, θ_i	=	parameter vectors
Θ_e, θ_{ie}, b_{0e}	=	parameter estimates
$\tilde{\Theta}$, $\tilde{\theta}_i$, $\tilde{b}_0$	=	parameter errors
ρ	=	density of air

I. Introduction

AEROELASTIC systems exhibit a variety of phenomena including instability, limit cycle, and even chaotic vibration.[1−3] Active control of aeroelastic instability is an important problem. Researchers have analyzed the stability properties of aeroelastic systems and designed controllers for flutter suppression.[4−24] Robust aeroservoelastic stability margins using μ method have been obtained.[4] Digital adaptive control of a linear aeroservoelastic model has been considered.[5] At the NASA Langley Research Center a benchmark active control technology (BACT) wind-tunnel model has been designed, and control algorithms for flutter suppression have been developed.[6−11] References 7 and 8 describe unsteady aerodynamic data and flutter instability for the BACT project model. The classical and minmax methods have been used to derive robust flutter control systems.[9] Robust passification techniques have been used in Ref. 10 for control. Gain-scheduled controllers have been designed in Ref. 11. Neural and adaptive control of a transonic wind-tunnel model have been considered.[12,13] For an aeroelastic apparatus tests have been performed in a wind tunnel to examine the effect of nonlinear structural stiffness, and control systems have been designed using linear control theory, feedback linearizing technique, and adaptive control strategies.[14−22] A model reference variable structure adaptive control system for plunge displacement and pitch angle control has been designed using bounds on uncertain functions.[18] This approach yields a high-gain feedback discontinuous control system. A backstepping adaptive design method for flutter suppression has been adopted in Refs. 19 and 21. In this approach the aeroelastic model has been represented in an output feedback form by suitable coordinate transformation, and output feedback adaptive laws have been derived. A robust flutter control system has been presented in Ref. 20 in which a high-gain observer is used for estimating the unmeasured states and the lumped uncertain function of the model for synthesis. For synthesis of this robust controller, precise measurement of the pitch angle and plunge displacement is required because the high gain is sensitive to the measurement noise. Based on the Euler–Lagrange theory, control of an aeroelastic model has been considered.[23] A suboptimal control law for flutter suppression using the state-dependent Riccati equation (SDRE) method has been designed.[24] The SDRE method is applicable to nonminimum phase systems as well, but requires the knowledge of the system parameters.

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The state variable and output feedback designs of Refs. 17–19 and 21 are based on direct adaptive control techniques, in which the controller parameters are updated directly for control. However, direct adaptive nonlinear design does not offer the freedom of choice of parameter update laws. Unlike direct adaptive controllers, estimation-based adaptive designs provide a significant level of modularity of controller–identifier pair. In the modular design one designs a stabilizing controller and separately derives gradient or least-squares-type identifiers for synthesis. However, for nonlinear systems, one cannot combine any stabilizing controller with any identifier for adaptive control. In fact, such an attempt fails for nonlinear systems because even for a small parameter error the trajectory of the system can escape to infinity in a finite time. Of course, this does not happen for the linear systems. Therefore, for the estimation-based adaptive design it becomes essential to design a strong controller for achieving stability in the closed-loop system. In this respect it has been found that input-to-state-stabilizing (ISS) controllers in the control module have a sufficient degree of robustness to parameter uncertainty and are appropriate for modular design.[25] In spite of the flexibility offered by the modular design, it appears from the literature that this design approach using ISS controller has not been attempted for the control of aeroelastic systems; therefore, it is of interest to explore the applicability of the modular design for nonlinear flutter control.

The contribution of this paper lies in the design of modular adaptive control systems for the control of an aeroelastic system. The chosen aeroelastic model[14–17] describes the nonlinear plunge and pitch motion of a wing. This model has been traditionally used for the theoretical as well as experimental analyses of two-dimensional aeroelastic behavior. The model has both plunge and pitch polynomial-type structural nonlinearities. It is assumed that all of the system parameters are unknown to the designer, but the sign of control effectiveness coefficient and a lower bound on its magnitude are known. A linear, quasi-steady aerodynamic model is used with the aeroelastic model; however, it is noted that one can extend this approach to the case of nonlinear aerodynamics as well. Two modular adaptive control systems are designed by treating the pitch angle or the plunge displacement as a controlled output variable. The control system includes a control module and an identifier. First the control module is designed to accomplish input-to-state stability. This is followed by the design of an "observer-like" identifier based on the gradient-type adaptation law for parameter estimation. The ISS controller guarantees boundedness even in the presence of time-varying parameters without adaptation. In the closed-loop system the ISS controller together with the identifier accomplishes stabilization of the aeroelastic system. Simulation results for various flow velocities and elastic axis locations are obtained, which show that the modular design is effective in flutter suppression in spite of the large parameter uncertainties.

II. Aeroelastic Model and Control Problem

The prototypical aeroelastic wing section is shown in Fig. 1. The governing equations of motion are provided in Refs. 14–17, which are given by

$$\begin{bmatrix} m_t & m_w x_\alpha b \\ m_w x_\alpha b & I_\alpha \end{bmatrix} \begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h(h) & 0 \\ 0 & k_\alpha(\alpha) \end{bmatrix} \begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix} \tag{1}$$

where L and M are the aerodynamic lift and moment. It is assumed that the quasi-steady aerodynamic force and moment are of the form

$$L = \rho U^2 b c_{l_\alpha} s_p \left[\alpha + (\dot{h}/U) + \left(\tfrac{1}{2} - a\right) b (\dot{\alpha}/U)\right] + \rho U^2 b c_{l_\beta} s_p \beta$$

$$M = \rho U^2 b^2 c_{m_\alpha} s_p \left[\alpha + (\dot{h}/U) + \left(\tfrac{1}{2} - a\right) b (\dot{\alpha}/U)\right] + \rho U^2 b^2 c_{m_\beta} s_p \beta \tag{2}$$

where c_{l_α} and c_{m_α} are the lift and moment coefficients per angle of attack and c_{l_β} and c_{m_β} are lift and moment coefficients per control

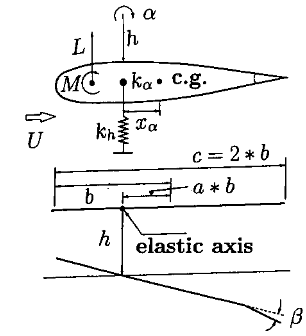


Fig. 1 Aeroelastic model.

surface deflection β. For illustrative purposes the function $k_\alpha(\alpha)$ and $k_h(h)$ are considered as polynomial nonlinearities of fourth and second degree, respectively. These are given by

$$k_\alpha(\alpha) = k_{\alpha 0} + k_{\alpha_1}\alpha + k_{\alpha_2}\alpha^2 + k_{\alpha_3}\alpha^3 + k_{\alpha_4}\alpha^4$$

$$k_h(h) = k_{h0} + k_{h_1} h^2$$

Defining the state vector $x = (x_1, \ldots, x_4)^T = (\alpha, h, \dot{\alpha}, \dot{h})^T \in R^4$, one obtains a state variable representation of Eq. (1) in the form

$$\dot{x} = \begin{bmatrix} 0_{2\times 2} & I_{2\times 2} \\ D_s & D_d \end{bmatrix} x + \begin{bmatrix} 0_{2\times 2} \\ G_0 \end{bmatrix} \begin{bmatrix} k_{n_\alpha}(\alpha) \\ k_{n_h}(h) \end{bmatrix} + \begin{bmatrix} 0_{2\times 1} \\ b_0 \end{bmatrix} \beta \tag{3}$$

where $\alpha k_\alpha = \alpha k_{\alpha 0} + k_{n_\alpha}$, $k_{n_\alpha} = \alpha(k_{\alpha_1}\alpha + k_{\alpha_2}\alpha^2 + k_{\alpha_3}\alpha^3 + k_{\alpha_4}\alpha^4)$, $h k_h(h) = k_{h_o} h + k_{n_h}$, $k_{n_h} = k_{h_1} h^3$, k_{ij} are constants, $G_0 = (g_{0ij})$ is a 2×2 constant matrix, $b_0 = (b_{01}, b_{02})^T$ (T denotes transposition), and 0 and I denote null and identity matrices of indicated dimensions. The matrices $D = (D_s, D_d) \in R^{2\times 4}$ and G_0 and b_0 are easily obtained from Eq. (1).

Assumption 1: It is assumed that the elements of matrices D, G_0, b_0; and k_{α_j}, $j = 0, 1, \ldots, 4$, and k_{h_j}, $j = 0, 1$, associated with the structural nonlinearities, are not known, but the sign of each element of b_0 is known, and

$$|b_{0k}| \geq b_{\mathrm{mk}} \tag{4}$$

($k = 1, 2$), where the lower bound b_{mk} is given.

We are interested in the design of modular adaptive control systems for the stabilization of the system (3) under assumption 1. For the purpose of deriving the adaptive control laws, the pitch angle or the plunge displacement is chosen as a controlled output variable.

III. ISS Pitch Angle Control Law

In this section a control module is designed for the choice of the pitch angle as an output, and the plunge motion control system is designed later. Using a backstepping design technique described in Ref. 25 (pp. 186–205), an input-to-state stabilizing control law is derived. This ISS control law has robustness to even time-varying parameter uncertainty and accomplishes input-to-state stability in the closed-loop system. First, we introduce the definition of ISS from Sontag.[26]

Definition 1 (ISS): The system

$$\dot{\eta} = g(t, \eta, u)$$

where g is piecewise continuous in t and locally Lipschitz in $\eta \in R^n$ and the input $u \in R^m$ is said to be ISS if for any $\eta(0)$ and for any continuous and bounded $u(.)$ on $[0, \infty)$ the solution exists for all $t \geq 0$ and satisfies

$$\|\eta(t)\| \leq \mu[\|\eta(t_0)\|, t - t_0] + \gamma[\sup\|u(\tau)\|, t_0 \leq \tau \leq t]$$

$\forall t_0$ and t such that $0 \leq t_0 \leq t$, where $\mu(s, p)$ and $\gamma(s)$ are strictly increasing functions of $s \in R_+$ with $\mu(0, p) = 0$, $\gamma(0) = 0$, while μ is a decreasing function of p with $\lim_{p\to\infty}\mu(s, p) = 0$, $\forall s \in R_+$.

We are interested in controlling the pitch angle as well as stabilization of the plunge motion. Although one can design a controller to track any smooth pitch angle reference trajectory, the reference trajectory is assumed to be identically zero because here the interest is in regulating the pitch angle to zero. The design is completed in two steps.

Step 1: The derivative of $z_1 = \alpha$ is

$$\dot{z}_1 = x_3 \tag{5}$$

Let

$$z_2 = x_3 - w_1 \tag{6}$$

be a new coordinate, where w_1 is a stabilizing signal chosen as

$$w_1 = -c_{11} z_1 \tag{7}$$

and the gain $c_{11} > 0$. Using Eqs. (5–7), one has

$$\dot{z}_1 = -c_{11} z_1 + z_2 \tag{8}$$

The choice of a Lyapunov function $V_1 = z_1^2/2$ yields

$$\dot{V}_1 = -c_{11} z_1^2 + z_1 z_2 \tag{9}$$

Note that x_3 is not a control input; therefore, in the next step one selects the control input for the regulation of z_2. Here step 1 is simple because Eq. (5) has no uncertainty.

Step 2: Consider the differential equation for z_2, which is

$$\begin{aligned}\dot{z}_2 &= c_{11} x_3 + D_1 x + G_{01}\left(k_{n_\alpha}, k_{n_h}\right)^T + b_{01}\beta \\ &\doteq c_{11} x_3 + \psi_1(\alpha, h)\theta_1 + b_{01}\beta\end{aligned} \tag{10}$$

where D_i and G_{0i} denote the ith rows of matrices D and G_0, respectively,

$$\psi_1 = [\alpha, h, \dot{\alpha}, \dot{h}, \alpha^2, \alpha^3, \alpha^4, \alpha^5, h^3] \in R^9 \tag{11}$$

and for $i = 1, 2$,

$$\theta_i = \left[D_i, g_{0i1}k_{\alpha_1}, \ldots, g_{0i1}k_{\alpha_4}, g_{0i2}k_{h1}\right]^T \in R^9 \tag{12}$$

Note that the vector θ_i is unknown.

Let θ_{ie} and b_{0ie} be the estimates of θ_i and b_{0i}, $i = 1, 2$, respectively. For the regulation of z_2, consider a control law

$$\beta = b_{01e}^{-1}\bar{w}_2 - \mathrm{sgn}(b_{01}) b_{m1}^{-1}[c_{22} + s_2(x)]z_2 \tag{13}$$

where the gain $c_{22} > 0$,

$$\bar{w}_2 = -z_1 - \psi_1\theta_{1e} - c_{11}x_3 \tag{14}$$

and the nonlinear damping term $s_2(x) \geq 0$ is yet to be determined.

Let $\tilde{\theta}_i = \theta_i - \theta_{ie}$ and $\tilde{b}_{0ie} = b_{0i} - b_{0ie}$ be the parameter errors ($i = 1, 2$). Using the control law Eq. (13) in Eq. (10) and noting that $b_{01}b_{01e}^{-1} = 1 + \tilde{b}_{01}b_{01e}^{-1}$, one has

$$\dot{z}_2 = -z_1 + \psi_1\tilde{\theta}_1 + \tilde{b}_{01}b_{01e}^{-1}\bar{w}_2 - |b_{01}|b_{m1}^{-1}(c_{22} + s_2)z_2 \tag{15}$$

For the stability analysis of Eqs. (8) and (15), consider a composite Lyapunov function

$$V_2 = V_1 + \left(z_2^2/2\right) \tag{16}$$

Its derivative along the solution of z subsystem [Eqs. (8) and (15)] is

$$\dot{V}_2 = -c_{11}z_1^2 + z_1 z_2 + z_2\dot{z}_2 \tag{17}$$

Using Eq. (15) in Eq. (17) and noting that $|b_{01}| \geq b_{m1}$ according to assumption 1, one obtains

$$\dot{V}_2 \leq -c_{11}z_1^2 + z_2\left[\psi_1\tilde{\theta}_1 + b_{01e}^{-1}\tilde{b}_{01}\bar{w}_2 - (c_{22} + s_2)z_2\right] \tag{18}$$

Using Young's inequality,[25] one has

$$|z_2\psi_1\tilde{\theta}_1| \leq \kappa z_2^2\left\|\psi_1^T\right\|^2 + \left(\|\tilde{\theta}_1\|^2/4\kappa\right) \tag{19}$$

$$\left|z_2\tilde{b}_{01}b_{01e}^{-1}\bar{w}_2\right| \leq \kappa z_2^2\left|\bar{w}_2 b_{01e}^{-1}\right|^2 + \left(|\tilde{b}_{01}|^2/4\kappa\right) \tag{20}$$

for any real number $\kappa > 0$. Using Eqs. (19) and (20) in Eq. (18) gives

$$\begin{aligned}\dot{V}_2 \leq{}& -c_{11}z_1^2 - (c_{22} + s_2)z_2^2 + \kappa\left[\left\|\psi_1^T\right\|^2 + \left|b_{01e}^{-1}\bar{w}_2\right|^2\right]z_2^2 \\ &+ \left(\left\|\tilde{\theta}_1^T\right\|^2 + |\tilde{b}_{01}|^2\right)(4\kappa)^{-1}\end{aligned} \tag{21}$$

In view of Eq. (21), one chooses the nonlinear damping term s_2 as

$$s_2(x) = \kappa\left[\left\|\psi_1^T\right\|^2 + \left|b_{01e}^{-1}\bar{w}_2\right|^2\right] \tag{22}$$

The design parameter κ is suitably chosen to control the magnitude of β. Substituting Eq. (22) in Eq. (21) gives

$$\begin{aligned}\dot{V}_2 &= (1/2)(\mathrm{d}\|z\|^2/\mathrm{d}t) \leq -c_{11}z_1^2 - c_{22}z_2^2 + \left[\|\tilde{\theta}_1(t)\|^2 + |\tilde{b}_{01}(t)|^2\right](4\kappa)^{-1} \\ &\leq -c^*\|z\|^2 + \left[\|\tilde{\theta}_1(t)\|_\infty^2 + |\tilde{b}_{01}(t)|_\infty^2\right](4\kappa)^{-1}\end{aligned} \tag{23}$$

where $c^* = \min(c_{ii}, i = 1, 2)$, $z = (z_1, z_2)^T$, and for any function $l(\tau)$, $\tau \in [0, t)$ one defines $\|l(t)\|_\infty^2 = \sup\{\|l(\tau)\|^2, \tau \in [0, t)\}$. Let us assume that $\tilde{\theta}_1(t)$ and $\tilde{b}_{01}(t)$ are bounded, then solving Eq. (23) gives

$$\begin{aligned}\|z(t)\|^2 \leq{}& \|z(0)\|^2 e^{-2c^*t} \\ &+ 2\left[\|\tilde{\theta}_1(t)\|_\infty^2 + |\tilde{b}_{01}(t)|_\infty^2\right](4\kappa)^{-1}\int_0^t e^{-2c^*(t-\tau)}\,\mathrm{d}\tau \\ \leq{}& \|z(0)\|^2 e^{-2c^*t} + \left[\|\tilde{\theta}_1(t)\|_\infty^2 + |\tilde{b}_{01}(t)|_\infty^2\right](4c^*\kappa)^{-1}\end{aligned} \tag{24}$$

Using the relation $|d_1|^2 + |d_2|^2 \leq (|d_1| + |d_2|)^2$, Eq. (24) gives

$$\|z(t)\| \leq \|z(0)\|e^{-c^*t} + \left\{\left[\|\tilde{\theta}_1(t)\|_\infty^2 + |\tilde{b}_{01}(t)|_\infty^2\right](4c^*\kappa)^{-1}\right\}^{\frac{1}{2}} \tag{25}$$

By forming a vector f_z using the right-hand side of Eqs. (8) and (15), the z subsystem is written as $\dot{z} = f_z(t, z, \tilde{\theta}_1, \tilde{b}_{01})$, where the argument t indicates the dependence of f_z on h and $\dot{h}$. Then according to definition 1, Eq. (13) is an ISS control law for the z subsystem. In fact, Eq. (25) implies that the trajectory z is eventually confined to the ball B_{ρ^*}, where $B_{\rho^*} = \{z : \|z\| \leq \rho^*\}$, $\rho^* = \{(\|\tilde{\theta}_1\|_\infty^2 + |\tilde{b}_{01}|_\infty^2)(4c^*\kappa)^{-1}\}^{1/2}$. The size of the ball B_{ρ^*} can be reduced by choosing larger values of c^* and κ, but this requires larger control magnitude.

This completes the ISS control law design for the pitch angle control. Now the problem remains to design an identifier, which provides desirable properties to the parameter estimates θ_{1e} and b_{01e} for the stability in the closed-loop system.

IV. Passive Identifier

The design of a passive identifier is considered in this section. Define the parameter vector

$$\Theta = \left(b_0^T, \theta_1^T, \theta_2^T\right)^T \in R^{20}$$

and let the parameter error vector be $\tilde{\Theta} = \Theta - \Theta_e = (\tilde{b}_0^T, \tilde{\theta}_1^T, \tilde{\theta}_2^T)^T$, where Θ_e is the estimate of Θ. Then using the definitions of ψ_1 and θ_i in Eqs. (11) and (12), system (3) can be written as

$$\dot{x} = f(x) + F(x, \beta)\Theta \tag{26}$$

where $f(x) = (x_3, x_4, 0, 0)^T$ and

$$F(x, \beta) = \begin{bmatrix} & 0_{2\times 20} & & \\ \beta & 0 & \psi_1 & 0_{1\times 9} \\ 0 & \beta & 0_{1\times 9} & \psi_1 \end{bmatrix} \in R^{4\times 20} \tag{27}$$

Following Ref. 25 (p. 232), an observer is implemented as

$$\dot{\hat{x}} = [A_o - \lambda F(x,\beta)F^T(x,\beta)P](\hat{x} - x) + f(x) + F(x,\beta)\Theta_e \tag{28}$$

where $\hat{x}$ is an estimate of x, A_o is a Hurwitz matrix, and the symmetric positive definite matrix P is the unique solution of the Lyapunov equation

$$PA_o + A_o^T P = -I_{4\times 4} \tag{29}$$

It follows from Eqs. (26) and (28) that the observer error $\tilde{x} = x - \hat{x}$ is governed by

$$\dot{\tilde{x}} = \tilde{A}(x)\tilde{x} + F(x,\beta)\tilde{\Theta} \tag{30}$$

where

$$\tilde{A} = A_o - \lambda F(x,\beta)F^T(x,\beta)P \tag{31}$$

For the derivation of the adaptation law, consider a positive definite Lyapunov function

$$V_o = \tilde{\Theta}^T\Gamma^{-1}\tilde{\Theta} + \tilde{x}^T P\tilde{x} \tag{32}$$

where the weighting matrix $\Gamma = \mathrm{diag}(\gamma_{ii})$, $i = 1,\ldots,20$, and $\gamma_{ii} > 0$. Differentiating V_o along the solution of Eq. (30) gives

$$\dot{V}_o = -2\tilde{\Theta}^T\Gamma^{-1}\dot{\Theta}_e + \tilde{x}^T\left(PA_o + A_o^T P\right)\tilde{x} - 2\lambda\tilde{x}^T PFF^T P\tilde{x} + 2\tilde{x}^T PF\tilde{\Theta} \tag{33}$$

To cancel unknown parameter-dependent functions in Eq. (33), one chooses the update law as

$$\dot{\Theta}_e = \Gamma F^T(x,\beta)P\tilde{x} \tag{34}$$

Substituting Eq. (34) in Eq. (33) gives

$$\dot{V}_o = -\|\tilde{x}\|^2 - 2\lambda\|F^T P\tilde{x}\|^2 \le 0 \tag{35}$$

In view of Eq. (35), it follows that $\tilde{x}$ and $\tilde{\Theta}$ are bounded. The observer has been termed as a passive observer, because considering $\tilde{\Theta}$ and $F^T P\tilde{x}$ as an input and output, respectively, the system (30) can be shown to be a strictly passive system.[25]

This adaptation scheme can lead to division by zero because inverse of b_{01e} is needed in the control law. Therefore, it is necessary to modify the adaptation rule for the estimate b_{01e}. The modification is done using the projection operator. Let $\tau_1 = \gamma_{11}F^T_{(1)}(x,\beta)P\tilde{x}$, where $F^T_{(1)}$ denotes the first row of F^T. The update law of the form $\dot{b}_{01e} = \mathrm{Proj}(\tau_1)$ using the projection operator is given by

$$\dot{b}_{01e} = \tau_1\begin{cases}1, & b_{01e}\,\mathrm{sgn}(b_{01}) > b_{m1} \quad\text{or}\quad \tau_1\,\mathrm{sgn}(b_{01}) \ge 0\\ \max\left\{0, \left[\epsilon - b_{m1} + b_{01e}\,\mathrm{sgn}(b_{01})\epsilon^{-1}\right]\right\}, & b_{01e}\,\mathrm{sgn}(b_{01}) \le b_{m1} \quad\text{and}\quad \tau_1\,\mathrm{sgn}(b_{01}) < 0\end{cases} \tag{36}$$

where $\epsilon \in (0, b_{m1})$. The update laws for the remaining parameters given in Eq. (34) are retained.

Let the initial condition be such that $b_{01e}(0)\mathrm{sgn}(b_{01}) > b_{m1}$, the lower bound. Then it can be shown following Ref. 25 (pp. 232, 233) that even with the modified adaptation rule the identifier has the following properties:

$$(i)|b_{01e}(t)| \ge b_{m1} - \epsilon > 0,\ t \in [0,\infty)$$

$$(ii)\tilde{\Theta} \in L_\infty[0,\infty),\ \dot{\Theta}_e \in L_2[0,\infty)$$

$$(iii)\tilde{x} \in L_2[0,\infty) \cap L_\infty[0,\infty)$$

This identifier has desirable properties, and this permits modular design possibilities.

For the system (3), the pitch angle chosen as an output has relative degree two[27] because the second derivative of the output α depends on β. Therefore, there exist zero dynamics of dimension two. The zero dynamics define the residual motion of the system (1) when the pitch angle is identically zero. It is well known that for stability in the closed-loop system zero dynamics must be stable. For the aeroelastic model the flow velocity and the elastic axis locations are the key parameters. The effect of these parameters on stability of the zero dynamics has been examined in several studies,[15–19] and it has been found that the origin of the zero dynamics is asymptotically stable for a range of these parameters.

Consider a state transformation

$$\Phi : x = (\alpha, h, \dot{\alpha}, \dot{h})^T \longmapsto (z_1, z_2, \xi_1, \xi_2)^T$$

where Φ is an appropriate 4×4 nonsingular matrix, $\mu = -(b_{02}/b_{01})$, $z_1 = x_1 = \alpha$, $z_2 = \dot{\alpha} + c_{11}\alpha$, and

$$\xi_1 = h + \mu\alpha, \qquad \xi_2 = \dot{h} + \mu\dot{\alpha} \tag{37}$$

Then it is easily seen that ξ dynamics can be expressed as

$$\dot{\xi} = f_\xi(z,\xi) \tag{38}$$

where $\xi = (\xi_1,\xi_2)^T$, $x = \Phi^{-1}(z^T,\xi^T)^T$, and

$$f_\xi(z,\xi) = \begin{Bmatrix} \xi_2 \\ (\mu, 1)\left[D\Phi^{-1}(z^T,\xi^T)^T + G_0(k_{n\alpha}, k_{\mathrm{nh}})^T\right]\end{Bmatrix} \tag{39}$$

For obtaining the zero dynamics one sets $z = 0$ in Eq. (38) to yield

$$\dot{\xi} = f_\xi(0,\xi) \tag{40}$$

Then the following result can be stated.

Theorem 1: Consider the systems (8), (15), and (38) and suppose that $\xi = 0$ of the zero dynamics (40) is exponentially stable. Then there exists a region Ω surrounding $(z,\xi) = 0$ such that the trajectories beginning in Ω are bounded, and z and ξ tend to zero as $t \to \infty$. Therefore, in the closed-loop system x converges to zero. (For a proof see the Appendices.)

Theorem 1 assumes the exponential stability of the zero dynamics. It is seen later that the origin of the zero dynamics is globally asymptotically stable. For this case theorem 1 holds for arbitrarily large subset Ω.

Computing the function f_ξ, it can be shown that for $z = 0$ the zero dynamics can be written as

$$\ddot{h} = -a_1\dot{h} - a_2(k_{h0}h + k_{h1}h^3) \tag{41}$$

where

$$a_2 = \left(m_t + m_w x_\alpha c_{l_\beta} c_{m_\beta}^{-1}\right)^{-1}$$

$$a_1 = \left[c_h + \rho s_p U b\left(c_{l_\alpha} - c_{l_\beta} c_{m_\alpha} c_{m_\beta}^{-1}\right)\right]a_2$$

The parameter $c_{m\beta}$ is negative, which implies that $a_1 > 0$ and a_2 is also positive for the system parameters given in the Appendix. Thus the linearized system (41) is exponentially stable. Furthermore, because the plunge structural stiffness is such that

$$\left(k_{h_0} + k_{h_1}h^2\right)h > 0$$

for $h \ne 0$, one can show that the origin $(h = 0, \dot{h} = 0)^T$ of the nonlinear system (41) is globally asymptotically stable (g.a.s.). This is easily verified by choosing a positive definite Lyapunov function

$$V_h = \frac{1}{2}\dot{h}^2 + a_2 \int_0^h \left(k_{h_0} y + k_{h_1} y^3\right) \mathrm{d}y \tag{42}$$

The derivative of V_h along the solution of Eq. (41) can be shown to be

$$\dot{V}_h = -a_1 \dot{h}^2 \leq 0 \tag{43}$$

Because V_h is positive definite, $\dot{V}_h$ is negative semidefinite, and the largest invariant set in $\Omega_h = \{(h, \dot{h}) : \dot{V}_h = 0\}$ is zero; the LaSalle invariance theorem [26] establishes g.a.s. of the equilibrium point $(h = 0, \dot{h} = 0)$ of the zero dynamics equation (41).

Following the derivation of the control law, one observes that for the control of the pitch angle only the sign of b_{01} and a lower bound b_{m1} from assumption 1 are needed. As such, one can assume that the parameter b_{02} is completely unknown as well for α control.

V. Plunge Control

In this section the design of a control system using the plunge displacement as an output is considered. Because the design can be completed following the steps of the preceding section, only a brief derivation for the plunge control law design is presented.

For the design of an ISS controller, consider the plunge dynamics of interest given by

$$\dot{x}_2 = x_4, \qquad \dot{x}_4 = D_2 x + G_{02}\left(k_{n_\alpha}, k_{n_h}\right)^T + b_{02}\beta \tag{44}$$

Following the steps 1 and 2 of Sec. IV, one obtains the control law

$$\beta = b_{012e}^{-1}\bar{w}_2 - \operatorname{sgn}(b_{02}) b_{m2}^{-1}[c_{22} + s_2(x)] z_2 \tag{45}$$

where now one has $z_1 = x_2 = h$, $z_2 = x_4 - w_1$, $w_1 = -c_{11} z_1$, and

$$\bar{w}_2 = -z_1 - \psi_1 \theta_{2e} - c_{11} x_4, \qquad s_2(x) = \kappa\left(\|\psi_1\|^2 + \left|b_{02e}^{-1}\bar{w}_2\right|^2\right)$$

Here for simplicity symbols for certain functions identical to those used in Sec. IV have been retained; however, one must note that these functions as defined here differ from those of the preceding section.

Interestingly, in this modular design for plunge control one does not need to redesign the identifier, and one simply implements the observer and the update law already designed for the pitch angle control. However, because the control law uses b_{02e}^{-1} a minor modification in the update law is required to avoid division by zero. Now the projection operator is used to modify only the update law for b_{02e}, and the adaptation rules for the remaining parameters (including b_{01e}) are governed by Eq. (34).

Let $\tau_2 = \gamma_{22} F_{(2)}^T(x, \beta) P \tilde{x}$, where $F_{(2)}^T$ denotes the second row of F^T. The update law of the form $\dot{b}_{02e} = \operatorname{Proj}(\tau_2)$ using the projection operator is given by

$$\dot{b}_{02e} = \tau_2 \begin{cases} 1, & b_{02e}\operatorname{sgn}(b_{02}) > b_{m2} \quad \text{or} \quad \tau_2 \operatorname{sgn}(b_{02}) \geq 0 \\ \max\left\{0, \left[\epsilon - b_{m2} + b_{02e}\operatorname{sgn}(b_{02})\epsilon^{-1}\right]\right\}, & b_{02e}\operatorname{sgn}(b_{02}) \leq b_{m2} \quad \text{and} \quad \tau_2 \operatorname{sgn}(b_{02}) < 0 \end{cases} \tag{46}$$

This completes the design.

For obtaining the zero dynamics one sets $h = \dot{h} = 0$ in Eq. (3). Again eliminating β from the resulting equations, one finds that the zero dynamics in this case take the form

$$\ddot{\alpha} = -a_1 \dot{\alpha} - a_2 \alpha \tag{47}$$

where

$$a^* = \left(I_\alpha + b^2 c_{m_\beta} m_w x_\alpha c_{l_\beta}^{-1}\right)^{-1}$$

$$a_2 = \left[k_\alpha(\alpha) + \rho s_p b^2 U^2 \left(c_{l_\alpha} c_{m_\beta} c_{l_\beta}^{-1} - c_{m_\alpha}\right)\right] a^*$$

$$a_1 = \left[c_\alpha + \rho s_p U b^3 (0.5 - a)\left(c_{l_\alpha} c_{m_\beta} c_{l_\beta}^{-1} - c_{m_\alpha}\right)\right] a^*$$

The origin $(\alpha = 0, \dot{\alpha} = 0)$ of the zero dynamics is asymptotically stable if $a_1 > 0$ and

$$\left[k_{\alpha_0} + \rho s_p b^2 U^2 \left(c_{l_\alpha} c_{m_\beta} c_{l_\beta}^{-1} - c_{m_\alpha}\right)\right] a^* > 0$$

Furthermore, if $a_1 > 0$ and $a_2 \alpha > 0$ for $\alpha \neq 0$, similar to the preceding section, using a Lyapunov function

$$V_\alpha = \frac{a_1 \dot{\alpha}^2}{2} + \int_0^\alpha a_2(y)\,\mathrm{d}y$$

one shows that the origin of Eq. (47) is globally asymptotically stable. For the values of parameters U and a for which the origin of the zero dynamics is asymptotically stable, one concludes that in the closed-loop system the flutter suppression is accomplished.

Following the derivation of the plunge control law, one observes that for the control of the plunge motion only the sign of b_{02} and a lower bound b_{m2} from assumption 1 are needed. As such, one can assume that the parameter b_{01} is completely unknown as well for the purpose of the design of the plunge control system.

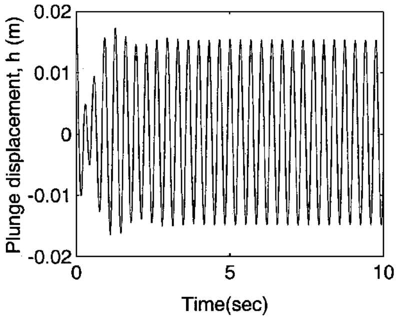
VI. Simulation Results

This section presents the results of simulation. The closed-loop system using the pitch and plunge control system is simulated. The initial conditions chosen are $x(0) = [11.46\ \text{(deg)}, 0.02(m), 0, 0]^T$. The initial values of the parameter estimates are set to $\theta_{ie}(0) = 0, i = 1, 2$ for all simulations. The initial condition for b_{0e} is $b_{0e}(0) = 2b_0$ (twice the nominal value) or $b_{0e} = 0.25b_0$ (one-fourth of the nominal value). Such large uncertainties in the parameter estimates are rather unfavorable for the controller; however, these have been chosen to examine the controller performance. The control gains are chosen as $\kappa = 5$, $c_{11} = c_{22} = 2$, $\lambda = 4$, and $\Gamma = 10^6 I_{20\times 20}$. The Hurwitz matrix A_o is chosen to have all of its eigenvalues at -5. Simulation results are presented for the two sets: (S1) [$U = 18$ (m/s), $a = -0.8$], and (S2) [$U = 20$ (m/s), $a = -0.6847$) of the flow velocity and the parameter a. For simulation, control saturation has been introduced to limit the surface deflection within 30 deg.

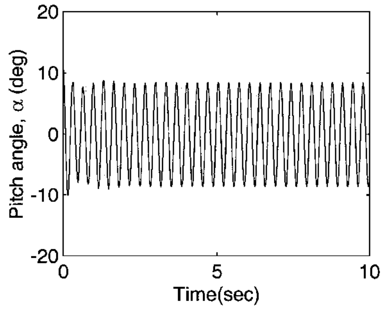
For the case (S1) the complex open-loop poles and zero of the linearized system for α as an output are at $(1.53 \pm j15.48, -3.92 \pm j15.19)$ and $-1.38 \pm j19.36$, respectively. For case (S2) for plunge displacement as an output, the poles are at $(1.68 \pm j15.05, -3.94 \pm j14.89)$, and the zeros are at $(-0.33 \pm j11.20)$, respectively. For each case the open-loop system is unstable, but the system has stable zeros (minimum phase system). The open-loop response for each case shows existence of limit cycle. The plots for case S1 are shown in Fig. 2, which show that after an initial transient the pitch angle and the plunge displacement trajectories converge to limit cycles.

A. Case A: Adaptive α Control for S1

The closed-loop system including the ISS controller, observer, and adaptation law is simulated. It is assumed that $b_{0e}(0)$ is twice the nominal value of $b_0 = (-28.99, -8.43)^T$ for case S1. The value of the lower bound is $b_{m1} = 5$ [less than (1/5)th of $|b_{01}|$]. Responses are shown in Figs. 3a–3g. It is observed that the pitch angle is

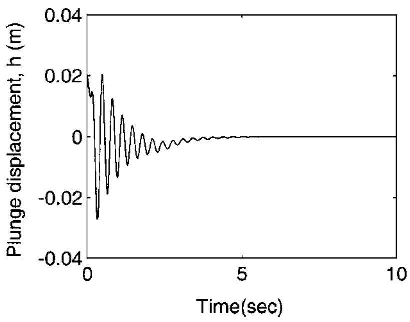


a) Plunge displacement (m)

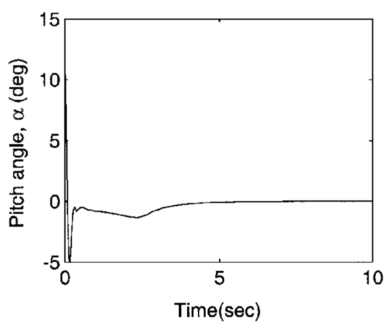


b) Pitch angle (deg)

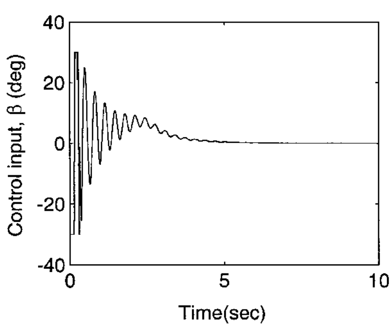
Fig. 2 Open-loop response: $a = -0.8$ and $U = 18$ m/s.



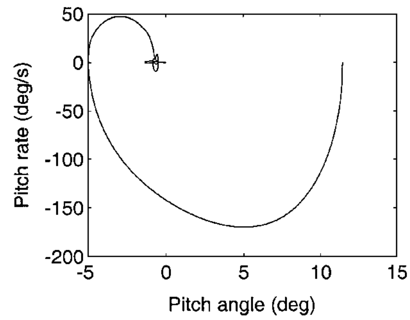
a) Plunge displacement (m)



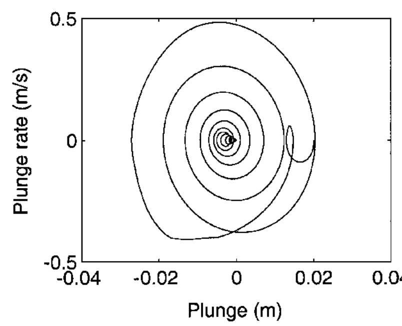
b) Pitch angle (deg)



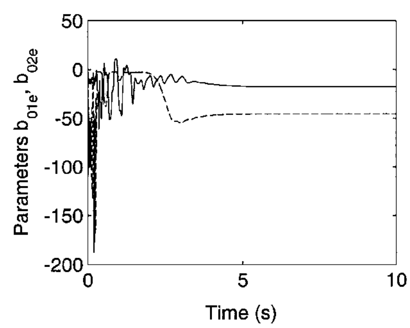
c) Control input β (deg)



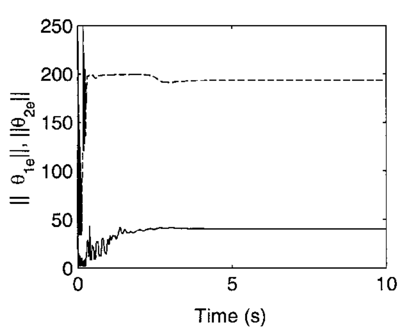
d) Phase plane plot α (deg) $-\dot{\alpha}$ (deg/s)



e) Phase plane plot h (m) $-\dot{h}$ (m/s)



f) Parameter estimates b_{01e} (——), b_{02e} (- - -)



g) Norm of parameter estimate vector $\|\theta_{1e}\|$ (——), $\|\theta_{2e}\|$ (- - -)

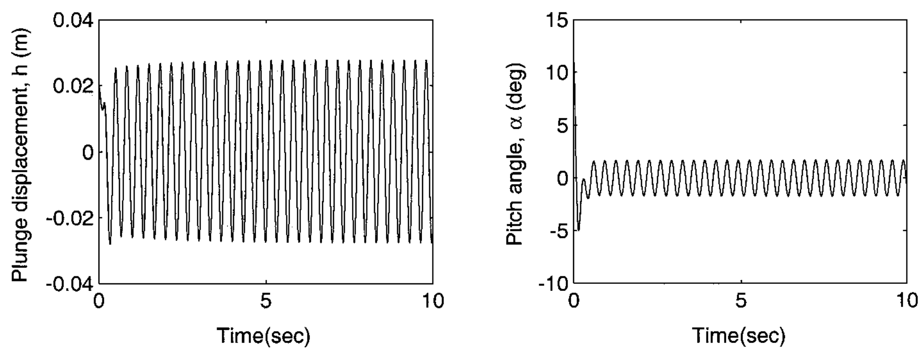
Fig. 3 Adaptive α control: $a = -0.8$, $U = 18$ m/s, $b_{0e}(0) = 2b_0$, $\theta_{ie}(0) = 0$, $i = 1,2$.

quickly controlled, and after initial oscillatory transient the plunge displacement also decays to zero. These oscillations in the h response are caused as a result of the complex zeros (the poles of the zero dynamics). Control saturation is observed very briefly. It is found that the projection operator avoids division by zero and keeps the estimate of $|b_{01e}| > 2.5$ because ϵ has been set to $b_{m1}/2$ for simulation. It is seen that all of the estimated parameters converge to constant values, but these values might not be the actual values [see the proof of theorem 1 and Eq. (A7) in the Appendix for an explanation]. However, for the regulation of α to zero convergence of parameter estimates is not essential. Simulations using $b_{0e}(0) = 0.25b_0$ for cases S1 and S2 are also done. The responses are somewhat similar and are not shown here.

To examine the effect of adaptation of parameters, simulation is done for case S1 using the gains and initial conditions used to obtain Fig. 3, but with frozen parameter vector $\Theta_e(t) = \Theta_e(0)$ (adaptation switched off). It is observed that without parameter adaptation, flutter exists (Fig. 4). However, the ISS controller reduces the amplitude of oscillation of the pitch angle response and accomplishes boundedness of trajectories in the closed-loop system when there is no adaptation. But in this case the amplitude of the plunge oscillation has increased. Note that the plunge amplitude also can be reduced according to Eq. (25) by the choice of larger gains c_{ii} and κ. Larger gains will reduce the size of B_{ρ^*}, the set of ultimate boundedness, but will require larger control magnitude and excessive control saturation.

B. Case B: *h* Control for Condition S2

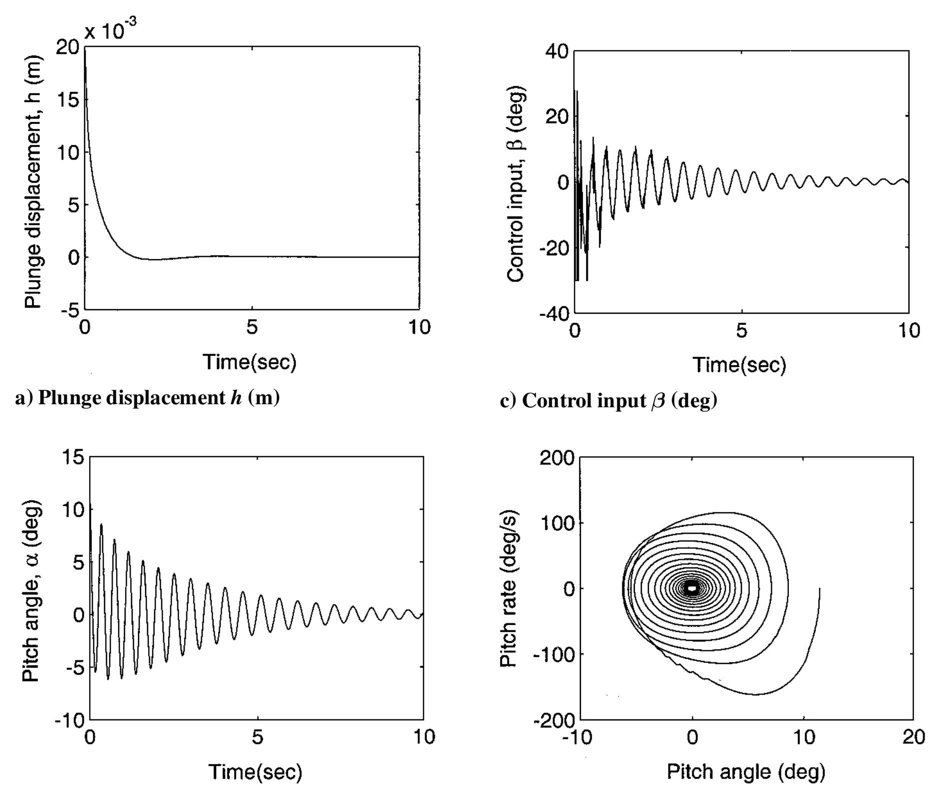
The complete closed-loop system for h control for case S2 is simulated. It is assumed that $b_{0e}(0) = 0.25b_0$ (one-fourth of the nominal value) and the lower bound is $b_{m2} = 1$, where $b_0 = (-43.83, -10.44)^T$. Thus the lower bound b_{m2} is about one-tenth of $|b_{02}| = 10.44$. The value of ϵ is selected as $b_{m2}/2$. Responses are shown in Fig. 5. It is observed that the plunge motion smoothly converges to zero, but the pitch angle has oscillatory convergent response. The zeros of the transfer function for the choice of the plunge displacement as an output are closer to the imaginary axis, and therefore the response time of the zero dynamics (α response) is slower. Furthermore, the zero dynamics in this case have a fifth-order polynomial nonlinearity compared to the zero dynamics corresponding to α control, which have only third-order nonlinearity.



a) Plunge displacement *h* (m)

b) Pitch angle α (deg)

Fig. 4 α control, adaptation switched off: $a = -0.8$, $U = 18$ m/s, $b_{0e}(0) = 2b_0$, $\theta_{ie}(0) = 0$, and $i = 1,2$.



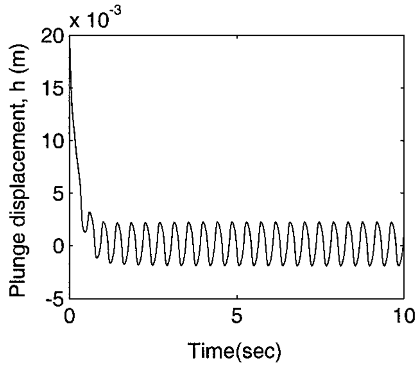
a) Plunge displacement *h* (m)

c) Control input β (deg)

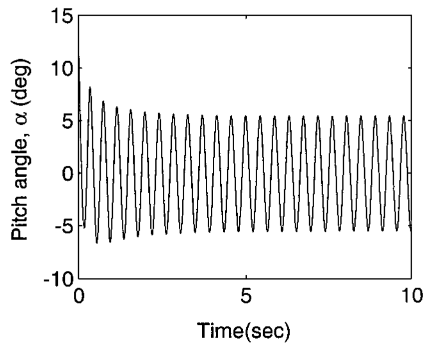
b) Pitch angle α (deg)

d) Phase plane plot α (deg) $-\dot{\alpha}$ (deg/s)

Fig. 5 Adaptive *h* control: $a = -0.6847$, $U = 20$ m/s, $b_{0e}(0) = b_0/4$, $\theta_{ie}(0) = 0$, and $i = 1,2$.



a) Plunge displacement h (m)



b) Pitch angle α (deg)

Fig. 6 Plunge displacement control, adaptation switched off: $a = -0.6847$, $U = 20$ m/s, $b_{0e}(0) = b_0/4$, $\theta_{ie}(0) = 0$, and $i = 1,2$.

Again, it is observed that the estimated parameters converge to some constant values and the projection operator is effective in limiting the parameter $|b_{02e}|$ away from $b_{m2} - \epsilon$.

Simulation is also done with frozen parameters (no adaptation) for the case S2 with $b_{0e}(0) = 0.25b_0$. Selected responses are shown in Figs. 6a and 6b. Again one observes that the controller fails to stabilize the system, and in the steady-state, limit cycles for the plunge and pitch motion persist. However, the ISS controller is effective in reducing the amplitudes of oscillations of both the plunge pitch responses. The open-loop system for this case (S2) (not shown here) has plunge and pitch oscillations of about 0.02 m and 10 deg, respectively.

Extensive simulation has been done for other values of U and a. It has been found that the adaptive control system is effective in stabilizing the closed-loop system. For the two conditions S_1 and S_2, it has been found that the zeros of the transfer function of the model under study for the choice of the plunge displacement as an output are closer to the imaginary axis compared to the case when the pitch angle is treated as an output. In this situation designing α controller will be preferable because the trajectories of the zero dynamics have a faster decay.

VII. Conclusions

In this paper adaptive control of a prototypical aeroelastic wing section with structural pitch and plunge nonlinearities using a single control surface was considered. It was assumed that only the sign of the coefficient of the control input and the lower bound on its absolute value were known, and the remaining parameters of the model were not known to the designer. Modular adaptive control systems using estimation-based design were derived for the control of the pitch angle and the plunge displacement. Each control system consisted of an input-to-state stabilizing control law, an observer, and a parameter adaptation law. In the closed-loop system asymptotic stabilization of the plunge and pitch motion was accomplished in spite of large uncertainties in the system parameters. It was shown by simulation that the closed-loop system without parameter adaptation failed to suppress flutter, but the input-to-state stabilizing controller was effective in keeping the pitch and plunge responses bounded when there was no adaptation.

The adaptive designs in literature assume quasi-steady aerodynamics. However, a realistic model must include unsteady aerodynamics as well. Although the backstepping design is applicable to such models, a new design is necessary because of the presence of additional state variables associated with the unsteady aerodynamics. The design of adaptive controllers for these aeroelastic systems is indeed an interesting research problem, which remains to be solved.

Appendix A: Proof of Theorem 1

In the closed-loop system it follows from Eq. (25) and boundedness of $\tilde{\Theta}$ [property (ii) of the identifier] that z is bounded. Of course boundedness of z implies that α and $\dot{\alpha}$ are bounded. Now it is shown that ξ is also bounded. Because $\xi = 0$ of the zero dynamics [Eq. (40)] is exponentially stable, by the converse theorem of Lyapunov, there exists a function $V_\xi : \Omega_\xi \to R$ that satisfies the following inequalities (Ref. 28, pp. 180, 181):

$$r_1\|\xi\|^2 \le V_\xi(\xi) \le r_2\|\xi\|^2, \qquad \partial\left(\frac{V_\xi}{\partial\xi}\right) f_\xi(z,\xi) \le -r_3\|\xi\|^2$$

$$\left\|\frac{\partial V_\xi}{\partial \xi}\right\| \le r_4\|\xi\| \tag{A1}$$

for all $\xi \in \Omega_\xi \subset R^2$ and some positive constants r_i, $(i = 1, \ldots, 4)$. We shall be interested in the domain $\Omega = \{(z,\xi) : V(z,\xi) = \text{constant}, \xi \in \Omega_\xi\}$, where V is a Lyapunov function

$$V(z,\xi) = V_2(z) + V_\xi(\xi) \tag{A2}$$

Because $f_\xi(z,\xi)$ is a differentiable function, it is Lipschitz in z and for $(z,\xi) \in \Omega$ satisfies

$$\|f_\xi(z,\xi) - f_\xi(0,\xi)\| \le L_0\|z\| \tag{A3}$$

where $L_0 > 0$.

Using Eqs. (23) and (38), the derivative of V can be written as

$$\dot{V} \le -c^*\|z\|^2 + \left(\|\tilde{\theta}_1\|_\infty^2 + |\tilde{b}_{01}|_\infty^2\right)(4\kappa)^{-1} + \left(\frac{\partial V_\xi}{\partial \xi}\right)[f_\xi(z,\xi) - f_\xi(0,\xi) + f_\xi(0,\xi)] \tag{A4}$$

Using the inequalities (A1) and (A3), one has

$$\dot{V} \le -c^*\|z\|^2 - r_3\|\xi\|^2 + L_0 r_4\|\xi\|\|z\| + \left(\|\tilde{\theta}_1\|_\infty^2 + |\tilde{b}_{01}|_\infty^2\right)(4\kappa)^{-1}$$
$$\doteq -[\|z\|, \|\xi\|]L^*[\|z\|, \|\xi\|]^T + \left(\|\tilde{\theta}_1\|_\infty^2 + |\tilde{b}_{01}|_\infty^2\right)(4\kappa)^{-1} \tag{A5}$$

where

$$L^* = \begin{bmatrix} c^* & -r_4L_0/2 \\ -r_4L_0/2 & r_3 \end{bmatrix}$$

It is seen that L^* can be always made positive definite by choosing c^* sufficiently large. Let $\lambda_m(L^*)$ be the smallest eigenvalue of the positive definite matrix L^*. Define

$$\Omega_0 = \left\{(z^T,\xi^T)^T : \|(z^T,\xi^T)^T\| < \left[\left(\|\tilde{\theta}_1\|_\infty^2 + |\tilde{b}_{01}|_\infty^2\right)(4\lambda_m\kappa)^{-1}\right]^{\frac{1}{2}}\right\} \tag{A6}$$

By choosing κ sufficiently large, one can make Ω_0 small enough so that it is strictly contained in Ω. Thus, in view of Eqs. (A5) and (A6), along the trajectory in Ω, $\dot{V}_2$ is negative if (z,ξ) is not contained in Ω_0. Therefore, the trajectory (z, ξ) beginning in Ω is uniformly bounded. This also implies that h and $\dot{h}$ are bounded.

The rest of the proof is completed using the derivation of Ref. 25 (p. 216). Because all signals are bounded, according to Eq. (30), $\dot{\tilde{x}}$ is bounded. Because $\tilde{x} \in L_2$ [property (iii)], by Barbalat's lemma (Ref. 25, p. 491), $\tilde{x}$ tends to zero. By virtue of the smoothness of F in Eq. (30), $\ddot{\tilde{x}}$ is also bounded, and therefore $\dot{\tilde{x}}$ is uniformly continuous. Also, integral of $\dot{\tilde{x}}$ is finite. Then by the Barbalat's lemma one has that $\dot{\tilde{x}} \to 0$. Thus from Eq. (30), one then has

$$F(x, \beta)\tilde{\Theta} \to 0 \tag{A7}$$

Substituting for $b_{01e}^{-1}\bar{w}_2$ from Eq. (13) in (15) gives

$$\dot{z}_2 = -z_1 + \psi_1\tilde{\theta}_1 + \tilde{b}_{01}\beta - |b_{01e}|b_{m1}^{-1}(c_{22} + s_2)z_2 \tag{A8}$$

Noting that the third row of $F\tilde{\Theta}$ is $F_{(3)}\tilde{\Theta} = \psi_1\tilde{\theta}_1 + \tilde{b}_{01}\beta$, the derivative of V_2 along the solution of Eqs. (8) and (A8) is

$$\dot{V}_2 = -c_{11}z_1^2 - |b_{01e}|b_{m1}^{-1}(c_{22} + s_2)z_2^2 + z_2F_{(3)}(x, \beta)\tilde{\Theta} \tag{A9}$$

Because $F\tilde{\Theta}$ converges to zero, the derivative of V_2 asymptotically tends to a negative definite function of z, which implies that z tends to zero (see Ref. 25, lemma B.8, p. 496). Now, invoking the exponential stability of the zero dynamics one concludes that ξ converges to zero. Thus x tends to zero. As x tends to zero, $F(x, \beta)$ also converges to zero. Therefore, according to Eq. (A7), $\tilde{\Theta}$ does not necessarily converge to zero. For the convergence of the parameters to their actual values, $F(x, \beta)$ must have sufficiently rich signals. This is possible if the input β is rich enough, that is, it contains signals of variety of frequencies. However, for the purpose of control the convergence of parameters is not required. This completes the proof of theorem 1.

Appendix B: System Parameters

The system parameters are as follows[15,22]:

$$b = 0.135\ \text{m}, \qquad m_w = 2.049\ \text{kg}, \qquad c_h = 27.43\ \text{Ns/m}$$

$$c_\alpha = 0.036\ \text{Ns}, \qquad \rho = 1.225\ \text{kg/m}^3, \qquad c_{l\alpha} = 6.28$$

$$c_{l\beta} = 3.358, \qquad c_{m\alpha} = (0.5 + a)c_{l\alpha}, \qquad c_{m\beta} = -0.635$$

$$m_t = 12.387\ \text{kg}, \qquad I_\alpha = 0.0517 + m_w x_\alpha^2 b^2\ \text{kg}\cdot\text{m}^2$$

$$x_\alpha = [0.0873 - (b + ab)]/b$$

$$k_\alpha = 6.861422(1 + 1.1437925\alpha + 96.669627\alpha^2 + 9.513399\alpha^3 - 727.66412\alpha^4)\ \text{N}\cdot\text{m/rad}$$

$$k_h = 2844.4 + 255.99h^2\ \text{N/m}$$

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